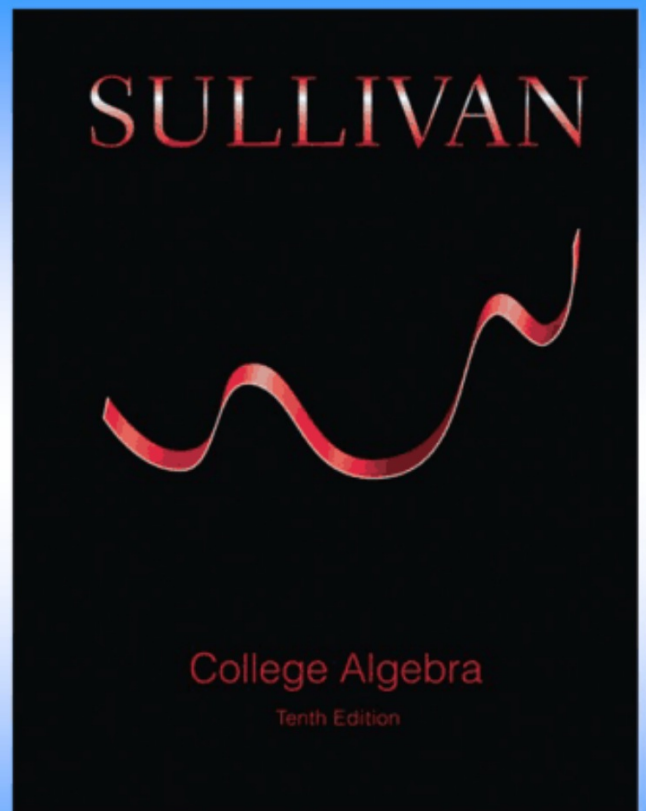


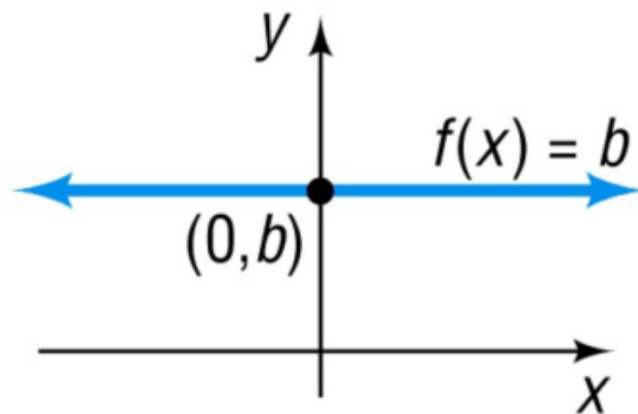
Chapter 3

Section 4 Library of Functions and Piecewise Functions



Constant Function

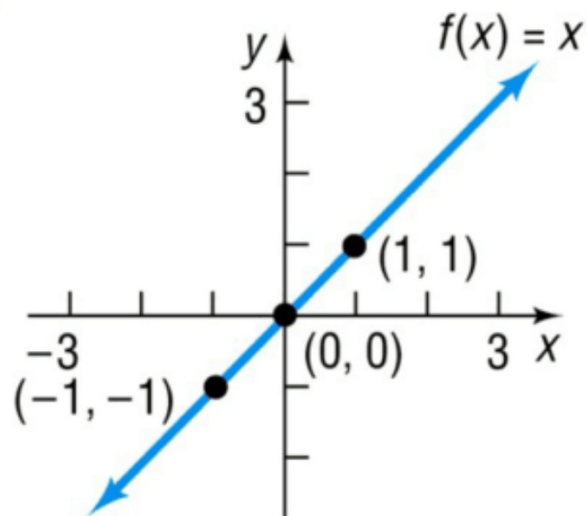
$$f(x) = b \quad b \text{ is a real number}$$



Identity Function

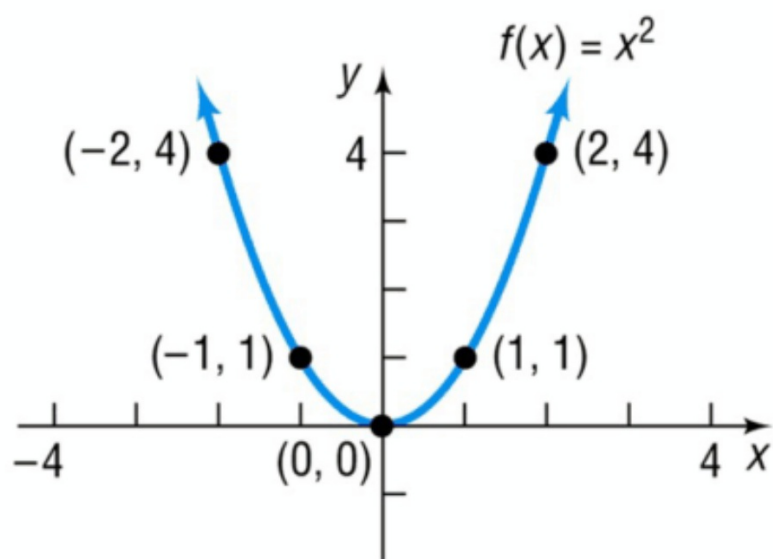
$$y = x$$

$$f(x) = x$$



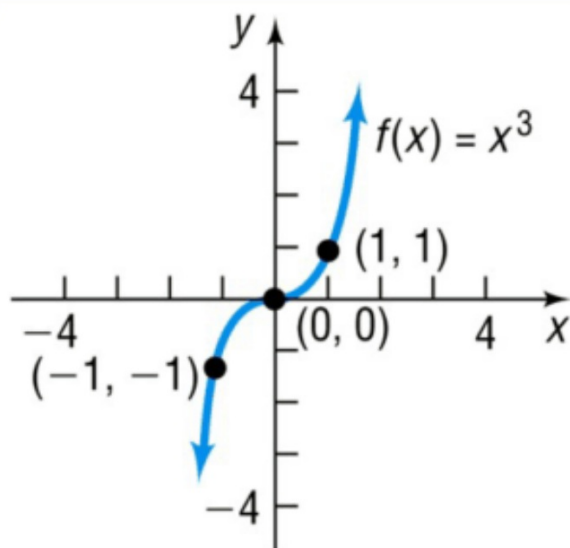
Square Function

$$f(x) = x^2$$



Cube Function

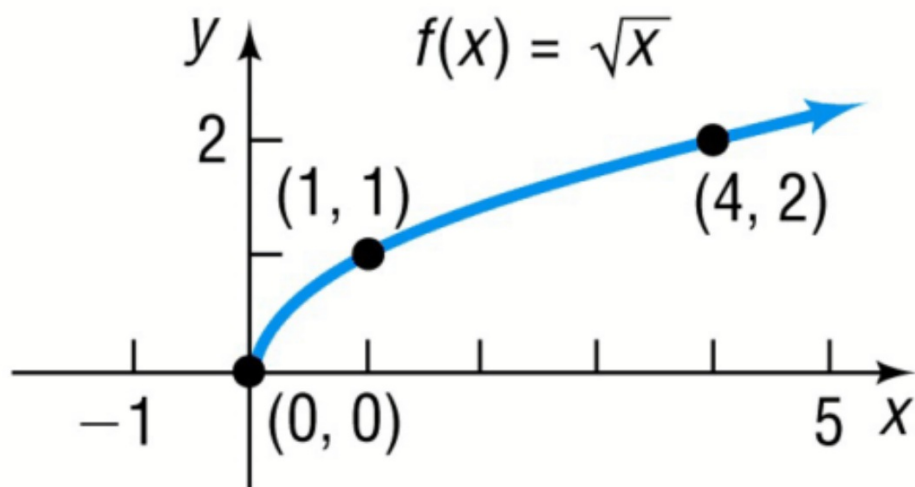
$$f(x) = x^3$$



Square Root Function

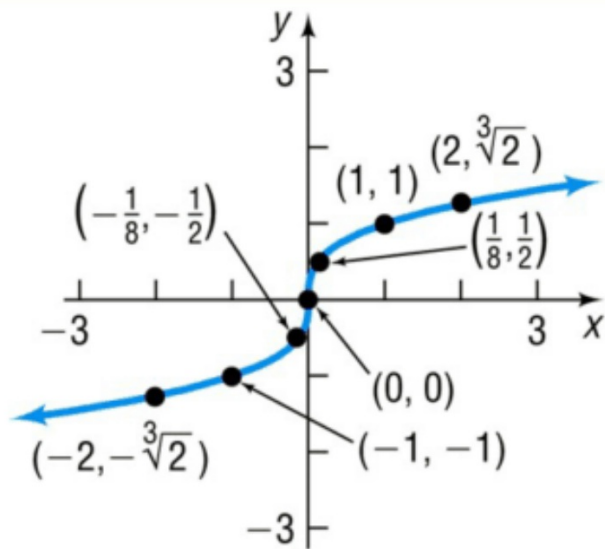
$$f(x) = \sqrt{x}$$

$$D: x \geq 0$$



Cube Root Function

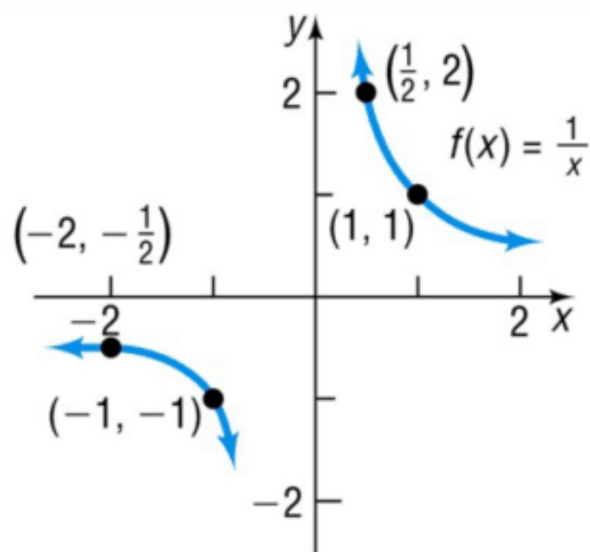
$$f(x) = \sqrt[3]{x}$$



Reciprocal Function

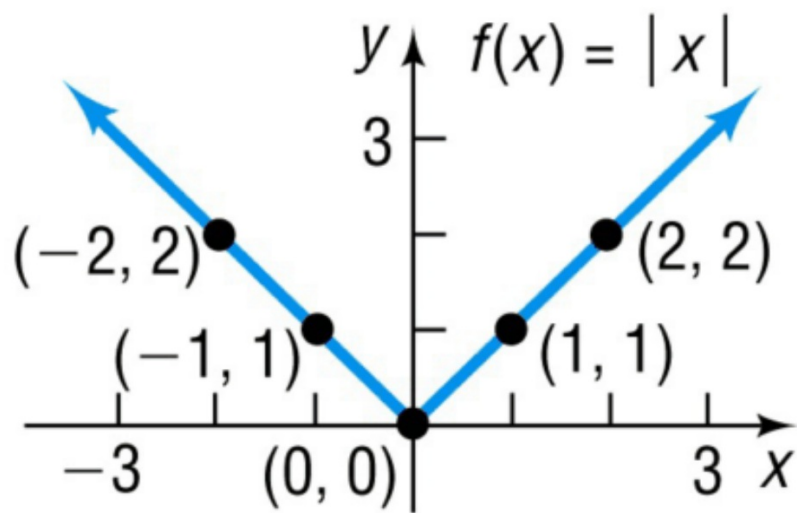
$$f(x) = \frac{1}{x}$$

$$D: x \neq 0$$



Absolute Value Function

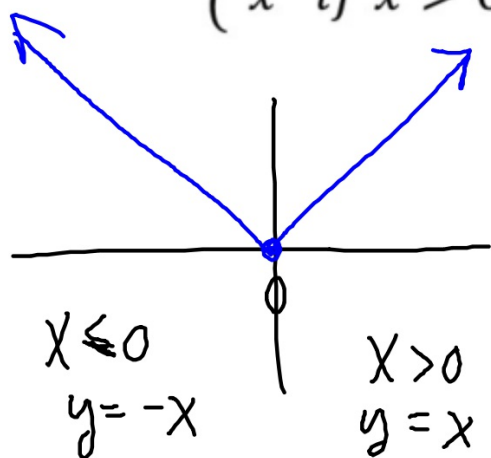
$$f(x) = |x|$$



Piecewise-Defined Functions

Piecewise-Defined functions are functions that have at least two different expressions defined on various intervals. Here is an example:

$$f(x) = \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$



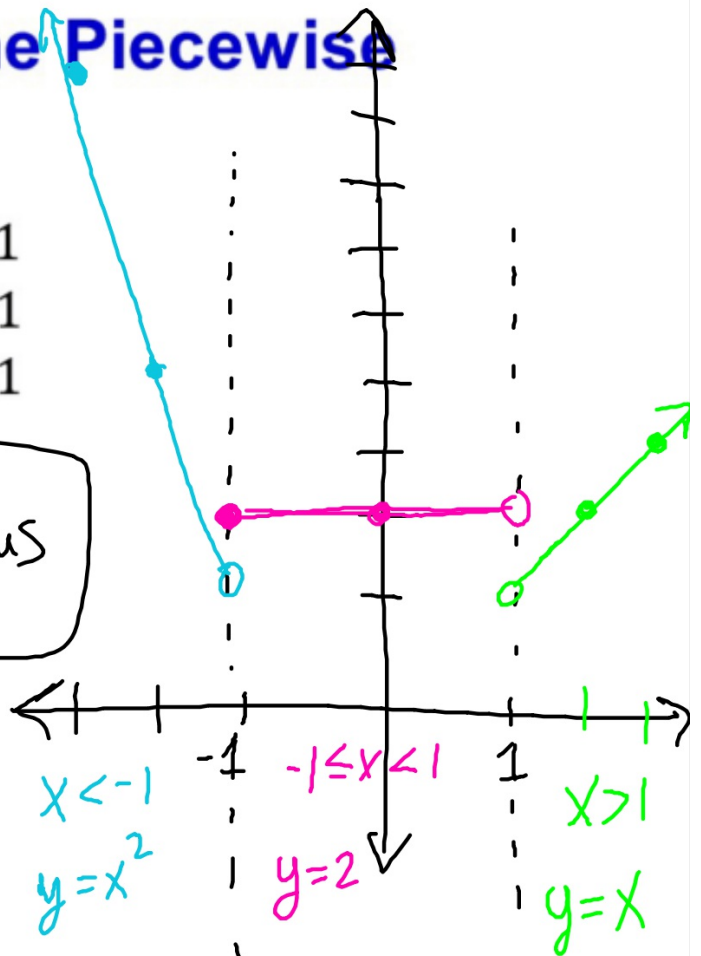
$$\begin{aligned} f(4) &= 4 & f(0) &= 0 \\ f(6) &= 6 \\ f(12) &= 12 \\ f(-3) &= 3 \\ f(-7) &= 7 \end{aligned}$$

Example: Graph the Piecewise Function

$$g(x) = \begin{cases} x^2 & \text{if } x < -1 \\ 2 & \text{if } -1 \leq x < 1 \\ x & \text{if } x > 1 \end{cases}$$

x	g(x)
-3	9
-2	4
-1	2
0	2
1	undefined
2	2
3	3

D: $x \neq 1$
discontinuous



Example

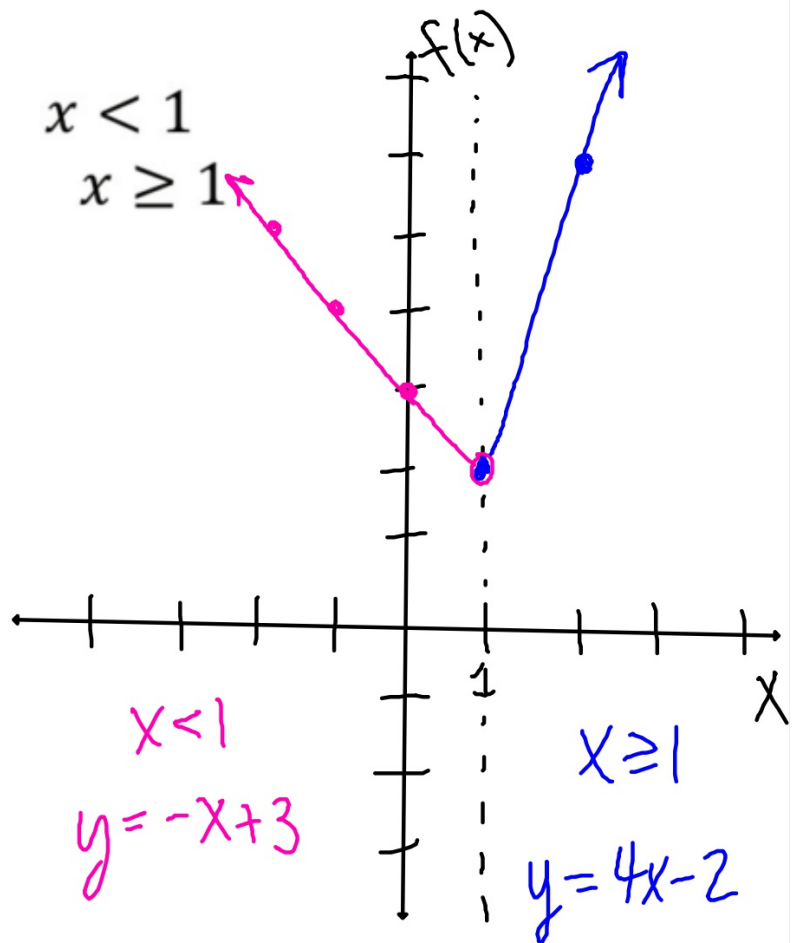
The function is defined as follows.

$$f(x) = \begin{cases} -x + 3, & x < 1 \\ 4x - 2, & x \geq 1 \end{cases}$$

- ✓ (a) Graph the function.
- (b) Locate any intercepts. Write your answers as ordered pairs. Answers: (0, 3)
- (c) Find the domain of the function in interval notation.
Answer: all reals $(-\infty, \infty)$
- (d) Find the range of the function in interval notation.
Answer: $y \geq 2$ $[2, \infty)$
- (e) Is the function continuous? Answer: Yes

$$f(x) = \begin{cases} -x + 3, & x < 1 \\ 4x - 2, & x \geq 1 \end{cases}$$

x	f(x)
-2	5
-1	4
0	3
1	2
2	6
3	10



Example

Analyzing a Piecewise-defined Function

The function f is defined as

$$f(x) = \begin{cases} -2x + 1 & \text{if } -3 \leq x < 1 \\ 2 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

- (a) Find $f(-2)$, $f(1)$, and $f(2)$.
(b) Determine the domain of f .
(c) Locate any intercepts.
(d) Graph f .
(e) Use the graph to find the range of f .
(f) Is f continuous on its domain?

(a) $f(-2) = -2(-2) + 1 = 5$
 $f(1) = 2$
 $f(2) = (2)^2 = 4$

(b) $x \geq -3 \dots [-3, \infty)$
(c) $(0, 1), (\frac{1}{2}, 0)$
(d) (f) No! $y > -1$
(e) $(-\infty, \infty)$

$$f(x) = \begin{cases} -2x + 1, & \text{if } -3 \leq x < 1 \\ 2, & \text{if } x = 1 \\ x^2, & \text{if } x > 1 \end{cases}$$

x	f(x)
-3	7
-2	5
-1	3
0	1
1	2
2	4

